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GRAVITY WAVES IN WATER CAUSED BY EXPLOSIONS

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ABSTRACT

This report discusses the gravity wave systems generated by explosions above on or under the water surface. A comparison is made between the theoretical results, obtained entirely from first principles, and those experimental results which are available. The agreement is reasonably good. The wave systems from large explosions are found to be unimpressive.

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GRAVITY WAVES IN WATER CAUSED BY EXPLOSIONS

INTRODUCTION

When an explosion occurs above or on the surface of water, the surface over a certain area received a downward impulse, denoted by the function $I(r)$, where r is the horizontal distance from the center. Experiments on blast indicate that $I(r)$ is zero if r exceeds the radius of the flame, i.e., 30 charge radii. At lesser radii, I increases with decreasing r, but the exact dependence is unknown and fortunately is not material to our discussion. All that matters is the surface integral of I, i.e., the total impulse given to the surface, and this can be estimated.

When an explosion occurs underwater, a cavity is formed. If the explosion is deep, the bubble pulsates rapidly and rises, until finally it breaks surface and escapes. If the explosion is at approximately a depth equal to the bubble radius, the surface film collapses at about the instant of maximum radius, and the water surface has the shape of a volcano. Such a configuration is the most likely one to produce the greatest waves. This contention is confirmed by experiment (see note number ADM/192/G.C., ARB, by the Road Research Laboratory; a copy of this note is in the writer's possession).

Accepting the statement made in the above paragraph, the optimum depth of detonation for various charge weights W lb of explosive may be computed in the following way. Imagine a spherical hole of radius h to be created in a sea of infinite extent, in such a way that the top of the sphere is just level with the surface of the sea. The work done in displacing water against hydrostatic plus atmospheric pressure is

$$W = 4\pi\rho g h^3(h+Z)/3 \tag{1}$$

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where Z is the head of water producing atmospheric pressure, and ρ is the density of water. Equate this work to 40% of E, the chemical energy of the explosion. Thus

$$W = 0.40\,E \tag{2}$$

The reason why 40% has been chosen is that the energy of an underwater explosion is known to be partitioned roughly as follows

(a) 30% wasted irreversibly in heating the water

(b) 40% in the pulsating motion of the bubble

(c) 30% radiated as a non-returning pressure pulse.

The energy released by a charge is about 1000 cal/gm. The calculated optimum wave system from a charge W lb is obtained when it is detonated at depth D ft given in the table below.

TABLE I

W	1/16	1/4	1.0	4.0	64	300	1000	4000	4×10^6
D	1.56	2.45	4.20	5.96	14.1	22.4	31.4	46.5	285
0.8D	1.25	1.96	3.36	4.77	11.3	17.9	25.1	37.2	228

The experimental results on 2-oz and 2-lb charges, obtained by the Road Research Laboratory and described in the note mentioned above, show that the optimum depths are about 0.8 of those calculated above, and that the wave height is quite sensitive to depth. Experiment also shows that the radius of the bubble, when it breaks, for a near-surface explosion is also about 0.8D. Hence the optimum depths in practice should be those corresponding with 0.8D of Table I.

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THE SCALING LAWS

The fundamental solution for cylindrically expanding infinitesimal waves in water of uniform depth d is

$$\phi = g \frac{\sin \sigma t}{\sigma} \frac{\cosh k(z+d)}{\cosh kd} J_o(kr)$$

$$\zeta = \cos \sigma t \, J_o(kr) \qquad (1)$$

where ϕ is the velocity potential, ζ is the surface elevation, d is the depth, and z is measured upwards from the free surface. Furthermore

$$\sigma^2 = gk \tanh kd \qquad (2)$$

Generalizing these results, we have that, corresponding with an initial surface elevation

$$\zeta_o = f(r), \qquad \phi_o = 0$$

$$\phi = g \int_0^\infty \frac{\sin \sigma t}{\sigma} \frac{\cosh k(z+d)}{\cosh kd} J_o(kr) \, kdk \int_0^\infty f(\alpha) \, J_o(k\alpha) \, \alpha d\alpha$$

$$\zeta = \int_0^\infty \cos \sigma t \, J_o(kr) \, kdk \int_0^\infty f(\alpha) \, J_o(k\alpha) \, \alpha d\alpha \qquad (3)$$

Similarly, corresponding with an initial surface impulse

$$\zeta_o = 0, \qquad \rho\phi_o = F(r)$$

$$\phi = \frac{1}{\rho} \int_0^\infty \cos \sigma t \, \frac{\cosh k(z+d)}{\cosh kd} J_o(kr) \, kdk \int_0^\infty F(\alpha) \, J_o(k\alpha) \, \alpha d\alpha$$

$$\zeta = -\frac{1}{g\rho} \int_0^\infty \sigma \sin \sigma t \, J_o(kr) \, kdk \int_0^\infty F(\alpha) \, J_o(k\alpha) \, \alpha d\alpha \qquad (4)$$

CASE I. Initial Surface Displacement

Here, we compare two explosions of charges W_1 and W_2. Writing

$$n^3 = W_2/W_1 \qquad (5)$$

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to solve the equation for the ratio of the linear scale of crater dimensions

$$\frac{h_2^3(h_2+Z)}{h_1^3(h_1+Z)} = n^3 \qquad (6)$$

and write

$$h_2/h_1 = m \qquad (7)$$

The scaling laws are

$$\alpha_2/\alpha_1 = d_2/d_1 = k_1/k_2 = r_2/r_1 = \sigma_1^2/\sigma_2^2 = t_2^2/t_1^2 = m$$

$$f_2(\alpha_2) = m f_1(\alpha_1) \qquad (8)$$

$$\zeta_2(m x_1 \sqrt{m}\, t) = m \zeta_1(x_1 t)$$

If a small model experiment is compared with a large explosion, h_1 is negligible compared with Z_1 but Z is negligible compared with h_2. Then

$$m = n^{3/4}(Z/h_1)^{1/4} \qquad (9)$$

The results to be expected from large explosions therefore scale up with those from small explosions in the ratio of the fourth root of the charge ratio, the corresponding distances and depths being in the same ratio.

The results to be expected from two small scale experiments may be approximated by assuming that h_1 and h_2 are both negligible compared with Z. Then

$$m = n$$

and the wave heights, distances and depths scale according to the linear dimensions of the charges.

CASE II. Initial Surface Impulse

Writing, as before,

$$n^3 = W_2/W_1,$$

the scaling laws are

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$$L_2 = nL_1$$

$$\alpha_2/\alpha_1 = r_2/r_1 = d_2/d_1 = k_1/k_2 = \sigma_1^2/\sigma_2^2 = t_2^2/t_1^2 = n$$

$$F_2(\alpha_2) \approx nF_1(\alpha_1)$$

$$\xi_2(nx_1\sqrt{n}\,t) = \sqrt{n}\,\xi_1(x_1t) \qquad (10)$$

Here L_2 and L_1 are the heights of detonation, d_2 and d_1 the depths of water.

The fact that the wave heights at corresponding distances are only in the sixth root of the ratio of the charge weights implies that only modest waves are to be expected from large surface explosions.

THE GROUP NATURE OF THE WAVE SYSTEMS

The wave systems generated by an explosion exhibit the familiar group characteristics. At any instant, one of the crests is greater than any of the others; similarly, one of the troughs. If the motion of this crest or trough is followed, it decreases in size and soon is dominated by its inside neighbor. The waves gradually pass through the group, and ultimately die out.

The main features of the system, i.e., the crest velocities, the group velocity, which wave is greatest at any instant, etc, can be calculated directly in terms of the size of the crater in an underwater explosion, and in terms of the flame radius for explosions above or on the surface.

According to the Cauchy-Poisson theory of surface waves (see Lamb's Hydrodynamics, p. 430), the surface elevation at ϖ, t, due to unit volume placed on the surface at 0, 0, is asymptotically

$$\xi_c = \frac{gt^2}{2^{5/2}\pi\rho\varpi^3}\cos(gt^2/4\varpi) \qquad (11)$$

Similarly, the surface elevation at ϖ, t, due to the application of unit impulse downward at 0, 0 is

$$\xi_1 = \frac{gt^3}{2^{7/2}\pi\rho\varpi^4}\sin(gt^2/4\varpi) \qquad (12)$$

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Numerical calculations from the accurate formulae show that these asymptotic formulae are accurate within 10% for all values of $\theta = gt^2/4\varpi$ greater than 8, and of course the accuracy increases with θ.

Assuming that the radius of the crater is h, the surface elevation at radius R at time t in deep water is

$$\xi(R,t) = \int f(r)\, \xi_c(\varpi,t)\, dS \qquad (13)$$

the integration being over the circle of radius $r = h$, while the initial crater depth at r is $f(r)$.

Similarly, for a surface impulse, the surface elevation at R, t is

$$\xi(R,t) = \int I(r)\, \xi_i(\varpi,t)\, dS \qquad (14)$$

Substituting (11) in (13), and (12) in (14) we deduce the following characteristics of the wave systems.

UNDERWATER EXPLOSIONS AT OPTIMUM DEPTH

The expanding wave system from the crater is led by a trough, the bottom of which moves according to

$$gt^2/R = 5 \quad \text{or} \quad R = 6.4\, t^2 \qquad (15)$$

(Actually, the wave system will be led by a crest, corresponding to the lip of the crater; for convenience, we call this wave the "swell".)

The n^{th} crest moves according to

$$R = gt^2/4(2n-1) \qquad (16)$$

so that the velocity of the n^{th} crest is

$$\dot{R} = gt/2(2n-1)$$

$$= \sqrt{gR/\pi(2n-1)} \qquad (17)$$

Hence the velocity of any crest or trough in deep water increases indefinitely with time, and the waves get larger.

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The n^{th} crest is greatest of all crests when

$$R = 2(2n-1)\,h \tag{18}$$

The velocity of the crest at this instant is

$$v = \sqrt{2gh/\pi} \tag{19}$$

The group velocity, i.e., the velocity of the region where the waves are greatest, is just half this quantity, namely

$$V = \sqrt{gh/2\pi} \tag{20}$$

The ratio of the greatest height of the m^{th} crest, to the greatest height of the n^{th} crest is

$$H_m/H_n = (2n-1)/(2m-1) \tag{21}$$

The motion of the p^{th} trough is given by

$$R = gt^2/8\pi p \tag{22}$$

and the trough is at its greatest size at

$$R = 4ph \tag{23}$$

Once a trough or crest has passed its maximum, it will decay like $k(R)\,R^{-2}$, where $k(R)$ is unity at the maximum, and increases rapidly to a limiting value about 3/2.

SURFACE EXPLOSIONS

The wave system is led by a crest, the velocity of the n^{th} crest being

$$v = gt/\pi(4n-3) \tag{24}$$

The n^{th} crest is the greatest of all crests at

$$R = gt^2/2\pi(4n-3) = (4m-3)a \tag{25}$$

$$t = (4n-3)\sqrt{2\pi a/g} \tag{26}$$

where a is the radius of the circle over which the impulse is delivered.

The velocity of the n^{th} crest at this instant is

$$v = \sqrt{2ga/\pi} \tag{27}$$

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The ratio of the greatest elevation in the m^{th} crest to the greatest elevation in the n^{th} crest is

$$H_m/H_n = (4n-3)/(4m-3) \tag{28}$$

The group velocity is

$$V = \sqrt{ga/2\pi} \tag{29}$$

The p^{th} trough is greatest when

$$R = (4p-1)a \tag{30}$$

$$t = (4p-1)\sqrt{2a/g} \tag{31}$$

Once a crest or trough has passed its maximum it decays like $\ell(R)\ R^{-5/2}$, where $\ell(R)$ is unity at the maximum but rapidly increases to a limiting value equal to about 3/2.

ABSOLUTE ESTIMATES AND COMPARISON WITH EXPERIMENT

UNDERWATER: Deep Water

The experimental evidence here refers to 2-oz and 2-lb charges of P.A.G. The theoretical and experimental wave heights are given for comparison in the table below. Two significant facts may be seen in the photographs of the experiments. (1) Waves after the second are not regular; the symmetry of the explosion is never perfect and all the products such as gas, spray and water drops damp the motion so much that only the first two waves are large. (2) The waves near the explosion break, as would be expected on theoretical grounds, since their height is greater than one sixth of their wave length.

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TABLE II

Charge	Optimum Depth	First Crest Theory	Second Crest Theory	Maximum Wave Height Theory		Maximum Wave Height Experimental	
2 oz PAG	1.50'	9" at 3.0'	3.0" at 9.0'	3.7" at 13'	1.0" at 30'	5" at 13'	1.6" at 30'
2 lb PAG	4.0'	24" at 8.0'	8.0" at 24.0'	13" at 28'	6" at 45'	7" at 28'	3.2" at 45'

The agreement is fair. We therefore feel reasonably certain that the theory will predict the waves from gadgets with an accuracy of about 50% either way, the estimates almost certainly being on the high side.

The table below gives the estimated wave heights (i.e., crest to trough) for gadgets 1000 tons and 10,000 tons.

TABLE III

H.E. Equiv. (tons)	Optimum Depth (feet)	1500'	2000'	3000'	4000'
1,000	190	40'	26'	11'	7'
10,000	340	---	94'	42'	23'

Surface Explosions

Here the experimental data are extremely meager. Extrapolating the data from Road Research Laboratory back to zero depth, gives the wave height at 13 ft from a 2-oz charge of P.A.G. on the surface as less than ½".

Let us calculate the wave heights from a 1-oz charge on the surface. If I_s is the total impulse given to the surface, then the second crest is the greatest crest of all when it is at 10 ft and the wave height in inches is.

$$H = 0.011\ I_s \qquad (32)$$

where I_s is in pounds weight seconds.

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An approximate value of I_s may be obtained from the knowledge that the explosion of a charge in free air may be regarded as caused by the release at the origin of a volume of gas at atmospheric pressure 20,000 times the charge volume. (This number is made up of 1000 for the gas products and 19000 for the irreversible shock wave heating of the air near the charge.) Hence the wave from a 1-oz charge on the surface may be roughly computed by the theory of sound waves caused by the release at the origin of 25 cubic feet of gas at atmospheric pressure. Then, the total impulse over the water surface should be

$$I_s = V \gamma p_o / 2c = 32 \text{ lb wt secs}$$

where V is the volume of gas (25 ft^3), γ is the ratio of the specific heats (1.4), p_o is atmospheric pressure (2100 lb/ft^2) and c is the velocity of sound (1180 ft/sec). Hence the height of the second crest is 1/3" at 10'. This equals the experimental figure, as far as our knowledge goes.

The wave heights (crest to trough) from 1000-ton and 10,000-ton gadgets exploded on the surface are given in the table

TABLE IV

H.E. Equivalent (tons)	1000'	1500'	2000'	3000'
1,000	30"	---	---	11"
10,000	48"	36"	25"	---

Explosions in Shallow Water

The theory given earlier cannot be expected to apply to shallow water. However, for purposes of comparison we give the theoretical results for deep water and compare them with some experimental results obtained by M. Shapiro and others obtained

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at Woods Hole (AM-793, UE-26). The volume of the crater for a charge exploded on the bottom at less than the optimum depth is assumed that of a hemisphere whose radius is equal to the depth.

TABLE V

Theory compared with Woods Hole data: 300-lb charge on bottom

Depth Ft	250'		500'	
	Theory	Expt	Theory	Expt
10	1.6"	1.8 ± 0.6"	0.4"	0.7 ± 0.2"
20	12"	2.6 ± 0.6"	3"	1.4 ± 0.3"

TABLE VI

Theory compared with Shapiro's data: 8-oz charge on bottom

Depth Ft	Theory at 6'	Expt at 6'	Theory 8'	Expt 8'
1	3.5"	7"	2.3"	4"

Scaling up Shapiro's data for a gadget of any H.E. equivalent exceeding 50 tons, on the bottom in 60 feet depth, we get 35' wave height at 360', 20' at 480', and about 5' at 1000'. These are definitely overestimates, because a wave will break when the wave height about equals the depth of water, and this condition is fulfilled at 250 to 300-feet distance.

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